# HEAT TRANSFER IN MHD COUETTE FLOW OF A RAREFIED GAS BETWEEN CONDUCTING WALLS

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**ABSTRACT**. An analysis of MHD Heat Transfer in Couette Flow of a viscous incompressible rarefield gas between electrically conducting walls is presented. Temperature jump boundary conditions are used to solve the differential equation. Expressions for temperature distribution and Nusselt number are obtained. The temperature profiles and Nusselt number are shown on graphs and conclusions are presented.

## NOMENCLATURE

$e$ = Electric field parameter.
$E$ = Eckert number
$M$ = Hartmann number
$Pr$ = Prandtl number
$T$ = Non-dimensional temperature
$u$ = Non-dimensional velocity component in $x$-direction
$y$ = Normal co-ordinate.
$\Gamma$ = Dimensionless temperature jump coefficient.
= $\xi_{t/2L}$
where $\xi_t$ = Temperature jump coefficient.
$\phi_l$ = Conductance ratio of the lower plate
$\phi_u$ = Conductance ratio of the upper plate
$\lambda$ = Rarefaction parameter.

## INTRODUCTION

The problem of MHD Couette flow between non-conducting walls has been discussed by Pai (1962). The corresponding flow between electrically conducting walls was analysed by Yen and Chang (1964). The heat transfer aspect of these problems were discussed by Soundalgekar (1968). In all these studies, the fluid was considered to be incompressible, electrically conducting and of normal density.

The flow of low density gases has also been discussed by Inman (1965), Soundalgekar (1967*a*, *b*, *c*). Inman discussed the channel flow between non-conducting walls whereas Soundalgekar (1967*a*) discussed the channel flow between

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conducting walls and the Couette flow between non-conducting (1967b) and conducting walls (1967c). The heat transfer aspect of the Couette flow and the channel flow was also discussed by Soundalgekar (1968). In all these papers, the flow was discussed with the help of the equations of the continuum media under slip flow and temperature jump boundary conditions.

The object of the present paper is to discuss the heat transfer aspect of the Couette flow between conducting walls under temperature jump boundary conditions and crossed fields.

## MATHEMATICAL ANALYSIS

For the fully developed flow, the energy equation is given in non-dimensional form (3) as

$$\frac{d^2T}{dy^2} + P_r.E\left[\left(\frac{du}{dy}\right)^2 + M^2(u-e)^2\right] = 0 \quad .. \quad (1)$$

and the temperature jump boundary conditions are (ref. 4)

$$\left.\begin{aligned} T(1) &= 1-2\Gamma\left(\frac{dT}{dy}\right)_{y=1} \\ T(-1) &= -1+2L\left(\frac{dT}{dy}\right)_{y=-1} \end{aligned}\right\} \quad .. \quad (2)$$

where the last term in (1) is written for the current density from Ohm's law. The expression for the velocity profile as derived in (1967c) is

$$u = \frac{\phi \text{ Cosh } My - \text{Sinh } My}{\text{Sinh } M(1+\lambda M \text{ Coth } M)} - \frac{\phi\,(\text{Coth } M+\lambda M)}{1+\lambda M \text{ Coth } M} \quad .. \quad (3)$$

where

$$\phi = \frac{\phi_u - \phi_l}{(\phi_u+\phi_l)M \text{ Coth } M+2}$$

and $\phi_u$, $\phi_l$, $M$, $e$ are defined by Soundalgekar (1967c)

Substituting for $u$ from (3) in (1), we have the solution of (1) in view of (2) as

$$T = C_1 y + C_2 - \frac{P_r.E}{B^2}\Big[\frac{\phi^2+1}{4} \text{ Cosh } 2\,My - \frac{\phi}{2} \text{ Sinh } 2\,My +$$

$$+2A_1(\text{Sinh } M)y + \frac{M^2A_1y^2}{4} - \phi \text{ Cosh } My\Big] \quad .. \quad (4)$$

where

$$C_1 = \frac{1}{2(1+2\Gamma)}\left[2+\frac{P_rE}{B^2}\{4A_1(2\Gamma M \text{ Cosh } M+\text{Sinh } M)-\right.$$

$$\left.-\phi(\text{Sinh } 2M+4\Gamma M \text{ Cosh } 2M)\}\right]$$

$$C_2 = \frac{P_rE}{B^2}\left[\frac{\phi^2+1}{4}\text{ Cosh } 2M+\Gamma M(\phi^2+1)\text{Sinh } 2M+\right.$$

$$\left.+\frac{M^2A_1^2}{2}+2\Gamma M^2A_1^2-2\phi A_1 \text{ Cosh } M-4\Gamma M\phi A_1 \text{ Sinh } M\right]$$

$$B = \text{Sinh } M(1+\lambda M \text{ Coth } M).$$

$$A_2 = (\phi+e\lambda M)\text{Cosh } M+(e+\phi\lambda M)\text{Sinh } M$$

The expressions for the Nusselt Number as derived in (Ref. 8) are

$$Nu_1 = \frac{4}{1-T_b}\left(\frac{dT}{dy}\right)_{y=1}$$

$$Nu_2 = \frac{4}{1+T_b}\left(\frac{dT}{dy}\right)_{y=-1} \quad .. \quad (5)$$

Hence from (4) and (5), we obtain

$$Nu_1 = \frac{4}{1-T_b}\left[C_1-\frac{P_rE}{B^2}\left\{\frac{M(\phi^2+1)}{2}\text{ Sinh } 2M-\phi M \text{ Cosh } 2M+\right.\right.$$

$$\left.\left.+2A_1M(\text{Cosh } M-\phi \text{ Sinh } M)+M^2A_1^2\right\}\right] \quad .. \quad (6)$$

and

$$Nu_2 = \frac{4}{1+T_b}\left[C_1-\frac{P_rE}{B^2}\left\{2A_1 M(\text{Cosh } M+\phi \text{ Sinh } M)-\right.\right.$$

$$\left.\left.-\frac{M(\phi^2+1)}{2}\text{ Sinh } 2M-\phi M \text{ Cosh } 2M-M^2A_1^2\right\}\right] \quad .. \quad (7)$$

The temperature profiles and the Nusselt number are shown in figures. 1–7.

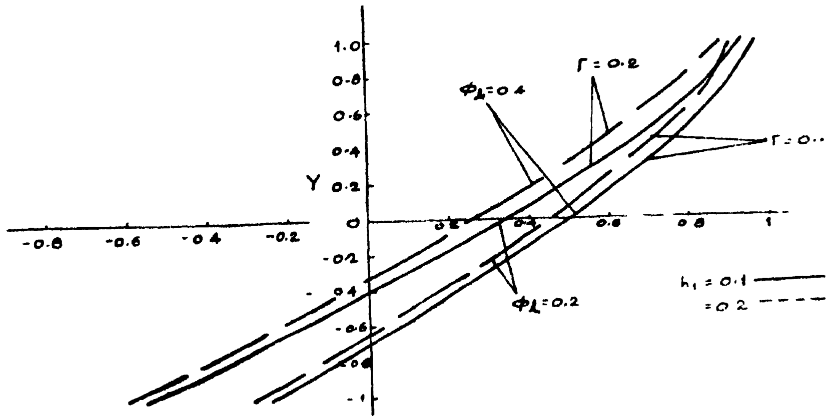


Figure 1. Temperature Profiles, $\varphi_u = 2$, M = 5, e = 0.2

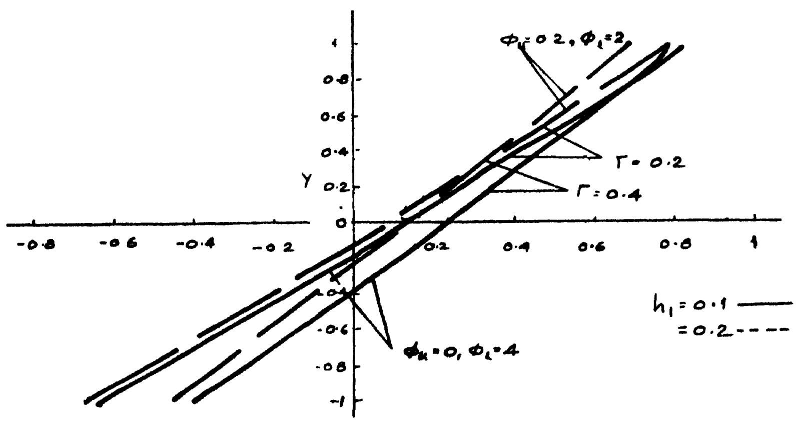


Figure 2. Temperature Profiles, M = 5, e = 0.2

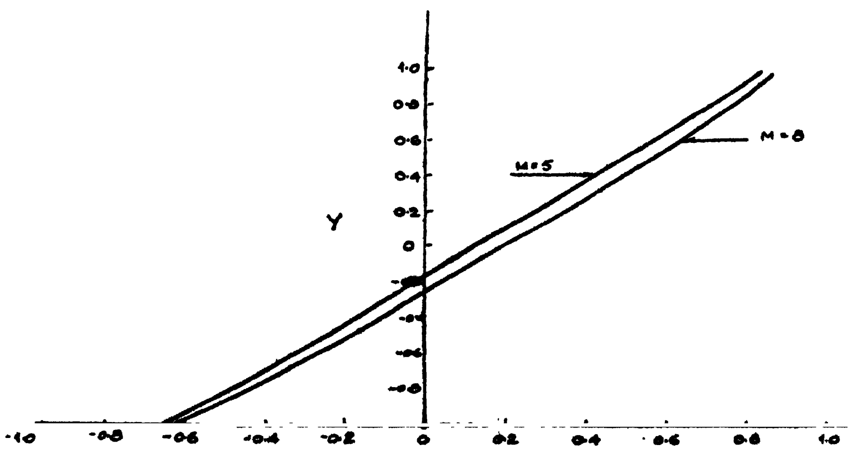


Figure 3. Temperature Profiles, $\varphi_u = 0.2$, $\varphi_l = 2$, $h_1 = 0.1$ $\Gamma = 0.2$, e = 0.2

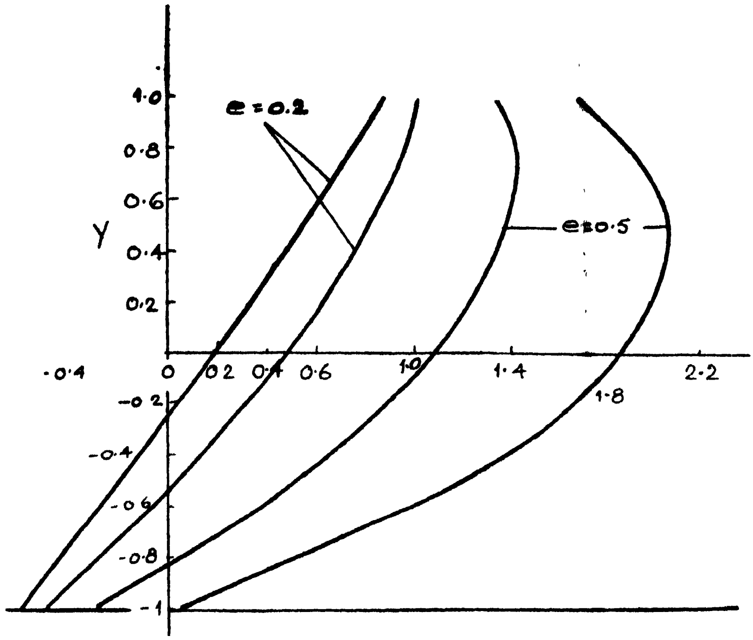


Figure 4. Temperature Profiles. $h_1 = 0.1$, $F = 0.2$ $\varphi_u = 0.2$, $\varphi_l = 0.2$ $M = 0$

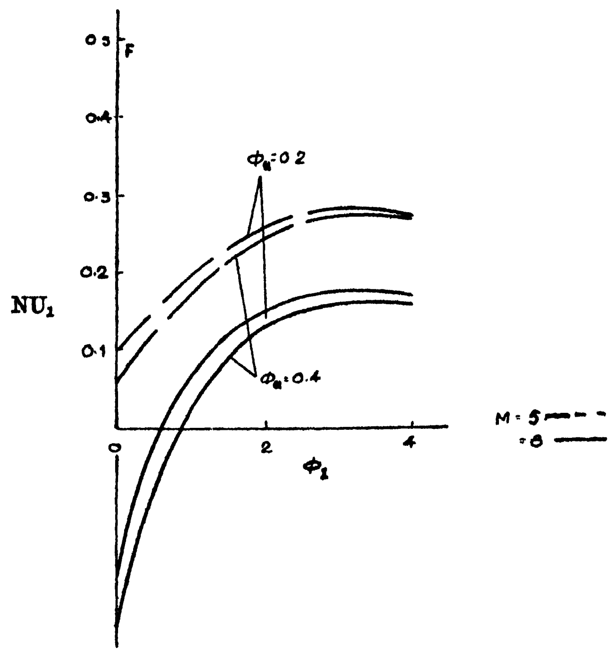


Figure 5. Nusselt number, $h_1 = 0.1$, $\Gamma = 0.4$, $e = 0.2$

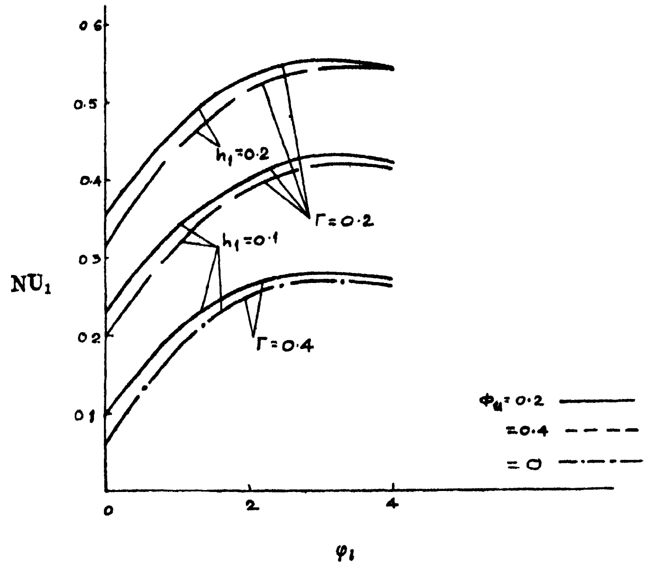


Figure 6. Nusselt number : $M = 5$, $e = 0.2$.

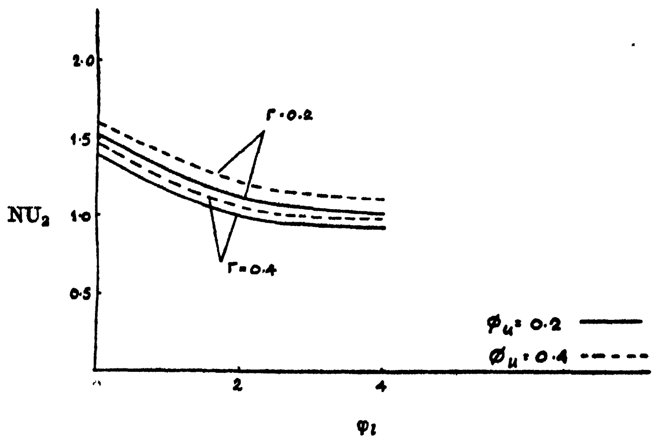


Figure 7. Nusselt number, $h_1 = 0.1$, $M = 5$, $e = 0.5$.

## CONCLUSION

In technological fields, the rate of heat transfer at the plate is expressed in terms of Nusselt number. Hence—

1. The Nusselt number at the upper plate ($Nu_1$) increases as the conductance ratio of the lower plate increases. But $Nu_1$ decreases as the Hartmann number and the conductance ratio of the upper plate increase.

2. The Nusselt number at the upper plate ($Nu_1$) increases as the rarefaction parameter ($\lambda$) increases for the same value of dimensionless temperature jump coefficient ($\Gamma$). But an increase in $\Gamma$ leads to a decrease in the Nusselt number $Nu_1$.

3. The Nusselt number at the lower plate decreases with increasing the conductance ratios of the upper, lower plates and $\Gamma$.

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